Well, Maybe It *Is* a Function!

Sequences and Functions

4.5

LEARNING GOALS

In this lesson, you will:

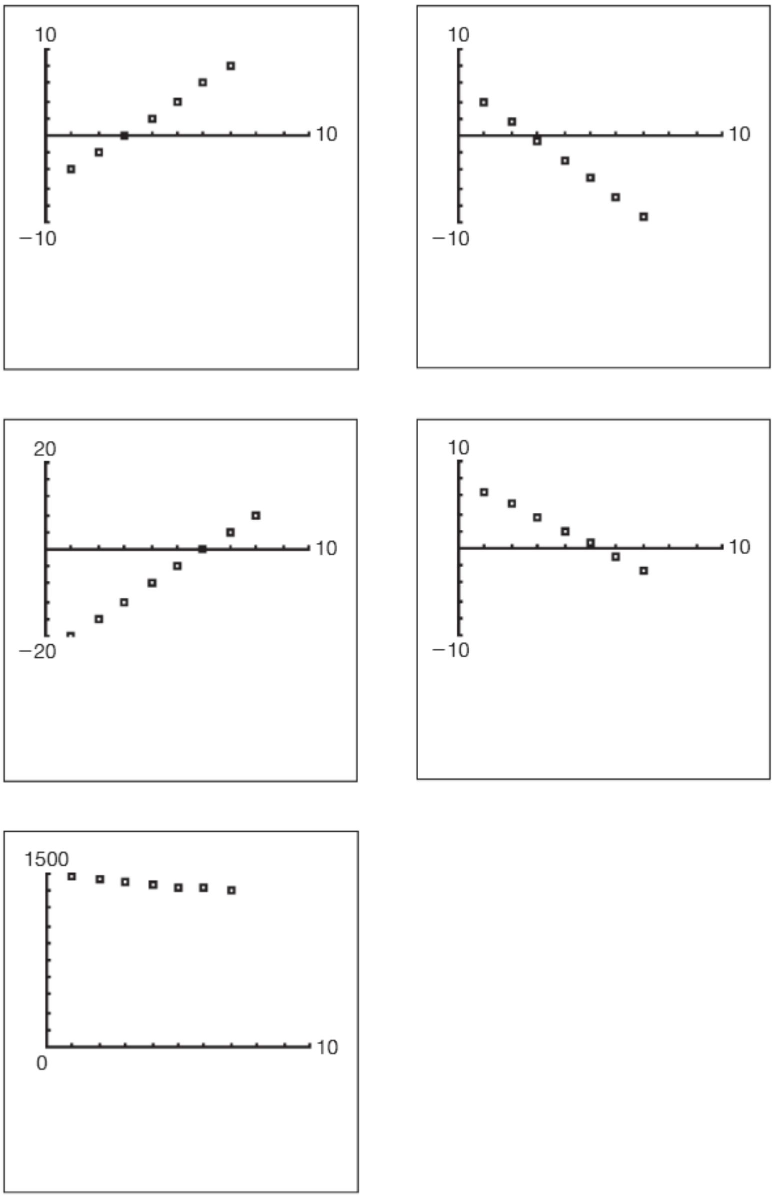
- Write an arithmetic sequence as a linear function.
- Make the connection between the graph of an arithmetic sequence, and the graph of a linear function.
- Write a geometric sequence as an exponential function.
- Make the connection between the graph of a geometric sequence, and the graph of an exponential function.
- Contrast an exponential function and a geometric sequence with a negative common ratio.

You might have heard the saying "If it looks like a duck and walks like a duck, it's probably a duck." The meaning is simple: if an object has some characteristics of something familiar, well, then it must be that familiar object, right? That seems pretty simple, but actually, it can be rather difficult. Attorneys and the law may counter with: "you can't judge a book by its cover."

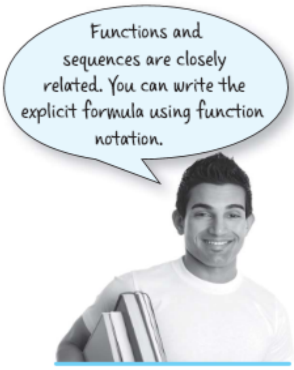
You have just encountered conjecture and proof through these two sayings. In mathematics, just like in law, more is needed than just a conjecture (a statement). A proof is needed. You will learn that in geometry, conjectures and proofs are a very important. But guess what? You're about to get an early dose of conjecture and proof!

PROBLEM 1 If It Looks Like a Function and Quacks Like a Function...

The graphs of the arithmetic sequences from Lesson 4.4, *Thank Goodness Descartes Didn't Drink Some Warm Milk!* are shown.



1. Identify the function family that represents the graphs of the arithmetic sequences shown. Do you think all arithmetic sequences belong to this function family? Explain your reasoning.



Consider the explicit formula for the arithmetic sequence shown in the first graph.

$$a_n = -4 + 2(n - 1)$$

You *can* write the explicit formula for the arithmetic sequence

$$a_n = -4 + 2(n - 1)$$

in function notation.

Statement	Reason
$a_n = -4 + 2(n - 1)$	Explicit Formula for Arithmetic Sequence
$f(n) = -4 + 2(n - 1)$	Represent a_n using function notation.
$f(n) = -4 + 2n - 2$	Distributive Property
$f(n) = 2n - 2 - 4$	Commutative Property
$f(n) = 2n - 6$	Associative Property

So $a_n = -4 + 2(n - 1)$ written in function notation is $f(n) = 2n - 6$.

2. In the Lesson 4.4, *Thank Goodness Decartes Didn't Drink Warm Milk!* you created graphic organizers that identified the explicit formulas for four arithmetic sequences. Rewrite each explicit formula in function notation.

a. Sequence E

$a_n = 4 + \left(-\frac{9}{4}\right)(n - 1)$

b. Sequence H

$a_n = -20 + 4(n - 1)$

c. Sequence K

$a_n = 6.5 + (-1.5)(n - 1)$

d. Sequence N

$a_n = 1473.2 + (-20.5)(n - 1)$

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3. Based on the formulas, identify the function family of these arithmetic sequences. Explain your reasoning.

4. What is the relationship between the common difference of an arithmetic sequence and the slope of a linear function?

5. Hank says that the y-intercept of a linear function is the same as the first term of an arithmetic sequence. Is he correct? Why or why not?

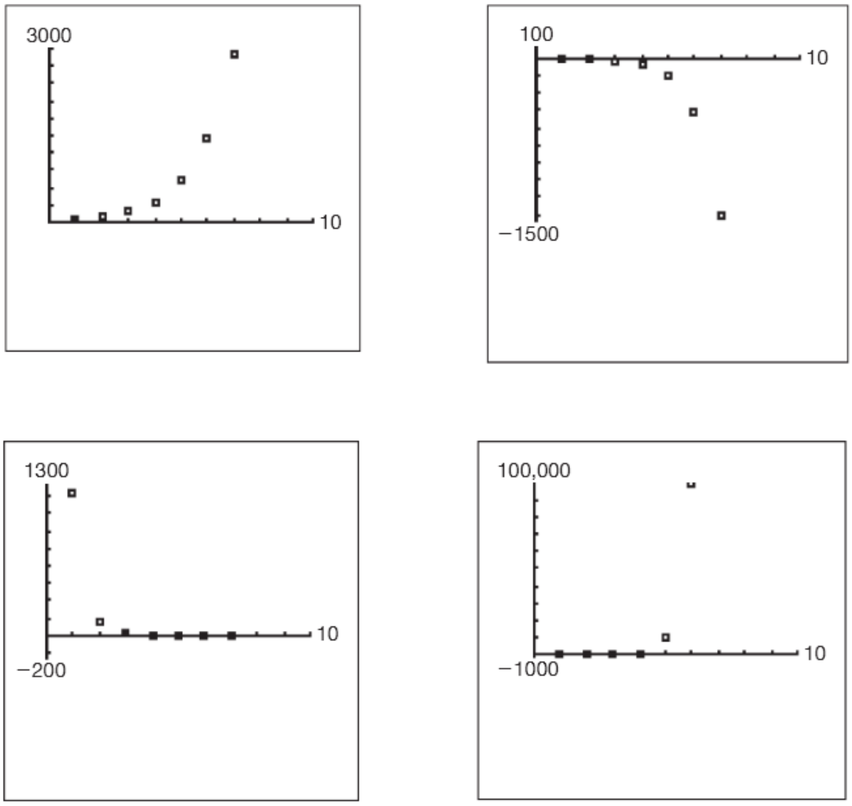
6. Represent the y-intercept of an arithmetic sequence algebraically.

7. Complete the table by writing each part of the linear function that corresponds to each part of the arithmetic sequence.

Arithmetic Sequence	Linear Function
$a_n = a_1 + d(n - 1)$	$f(x) = mx + b$
a_n	
d	
n	
$a_1 - d$	

PROBLEM 2 And If It Swims Like a Function and Smells Like a Function...

The graphs of the geometric sequences from Lesson 4.4, *Thank Goodness Descartes Didn't Drink Some Warm Milk!* are shown.



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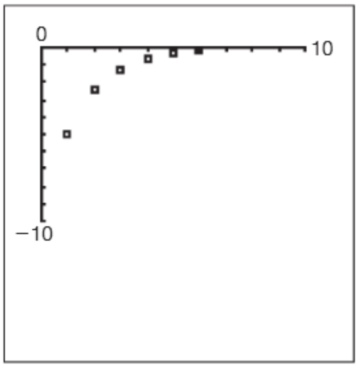

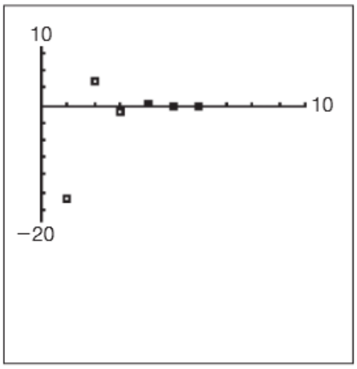

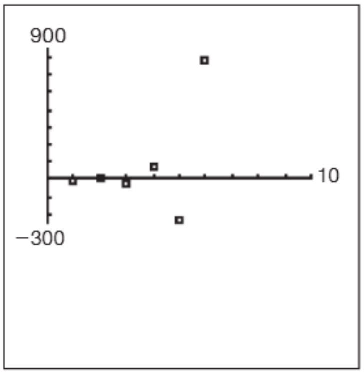

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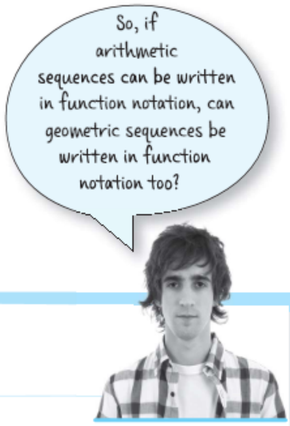
1. Do all of the graphs of the geometric sequences belong to the same function family? Why or why not?

Consider the explicit formula for the geometric sequence shown in the first graph.

$$g_n = 45 \cdot 2^{n-1}$$



You *can* write the explicit formula for the geometric sequence $g_n = 45 \cdot 2^{n-1}$ in function notation.

Statement	Reason
$g_n = 45 \cdot 2^{n-1}$	Explicit Formula for Geometric Sequence
$f(n) = 45 \cdot 2^n \cdot 2^{-1}$	Product Rule of Powers
$f(n) = 45 \cdot 2^{-1} \cdot 2^n$	Commutative Property
$f(n) = 45 \cdot \frac{1}{2} \cdot 2^n$	Definition of negative exponent
$f(n) = \frac{45}{2} \cdot 2^n$	Multiply.

So $g_n = 45 \cdot 2^{n-1}$ written as a function in function notation is $f(n) = \frac{45}{2} \cdot 2^n$.

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2. In the previous lesson, you created graphic organizers that identified the explicit formulas for six geometric sequences. Rewrite each explicit formula in function notation.

a. Sequence C

$g_n = -2 \cdot 3^{n-1}$

b. Sequence F

$g_n = 1234 \cdot 0.1^{n-1}$

c. Sequence I

$g_n = 1 \cdot 10^{n-1}$

d. Sequence J

$g_n = -5 \cdot \frac{1}{2}^{n-1}$

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e. Sequence M

$g_n = -16 \cdot \left(-\frac{1}{4}\right)^{n-1}$

f. Sequence P

$g_n = -4 \cdot (-3)^{n-1}$

3. Based on the formulas, do the geometric sequences belong to the same function family? Explain your reasoning.

4. What is the relationship between the common ratio of a geometric sequence and the base of the power in an exponential function?

5. What is the relationship between the first term of a geometric sequence and the coefficient of the power in an exponential function?

6. Complete the table by writing each part of the exponential function that corresponds to each part of the geometric sequence.

Geometric Sequence	Exponential Function
$g_n = g_1 \cdot r^{n-1}$	$f(x) = a \cdot b^x$
g_n	
$\frac{g_1}{r}$	
r	
n	

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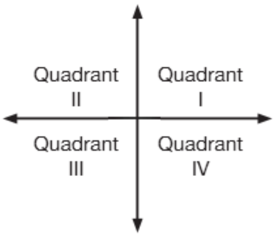
Talk the Talk

1. Complete each statement with always, sometimes, or never. Explain your reasoning for each statement.

   a. An arithmetic sequence can ____________ be represented as a linear function whose domain is the set of natural numbers.

   b. A geometric sequence can ____________ be represented as an exponential function whose domain is the set of natural numbers.

2. Determine whether the statement is true or false. If it is false, explain why it is false. Remember, the coordinate plane is split into four quadrants, as shown.



a. An arithmetic sequence will always begin in Quadrant 1.

b. An arithmetic sequence will never begin in Quadrant 3.

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c. A geometric sequence will sometimes begin in Quadrant 2.

d. A geometric sequence will always begin in Quadrant 4.

Be prepared to share your solutions and methods.